|  |
| --- |
|  |

***Integration***: The process of finding an anti-derivative or integral of a function is called integration. It is the inverse process of differentiation. If  be a function of  related with another function in such a way that



then



which is called an indefinite integral of  with respect to where,and  are called integrand, integral and constant of integration respectively.

And



which is called the definite integral of  from  to , and ‘’ is called the lower limit and ‘’ the upper limit of the definite integral.

***Fundamental Properties***:

1. .
2. 

where  is a constant.

***General rules of integration***:

1. . 2. .
2. . 4. .

5. . 6. .

1. . 8. .
2. . 10. .
3. . 12. .
4. . 14. .
5. . 16. .
6. . 18. .
7. . 20. .
8. . 22. .
9. . 24. .
10.  26. 

27.. 28. .

1. .
2. .
3. .
4. .
5. . 34. .

35..

36. .

***Illustrative Examples***:

**Problem-01**: **Exercise-01:** .

 Ans: 







where  is an integrating constant.

**Problem-02**: **Exercise-02**: 

 Ans: 





where  is an integrating constant.

**Problem-03:** **Exercise-03**: 

 Ans: .







where is an integrating constant.

**Problem-04:** **Exercise-04**: 

 Ans: .















where is an integrating constant.

**Problem-05**:  **Exercise-05**: .

 Ans:.









where is an integrating constant.

**Problem-06**:  **Exercise-06**:.

 Ans:.









where is an integrating constant.

**Problem-07**:  **Exercise-07**:.

 Ans:.













where is an integrating constant.

**Problem-08**:  **Exercise-08**: .

 Ans:.

















where is an integrating constant.

**Problem-09**:  **Exercise-09**:

 Ans: 











where is an integrating constant.

**Problem-10**: 











where is an integrating constant.

**Problem-11**:  **Exercise-09**: 

 Ans: .

















where is an integrating constant.

**Problem-12**: 













where is an integrating constant.

**Problem-13**:  **Exercise-10**: .

 Ans:.













where is an integrating constant.

**Problem-14:** **Exercise-11:**

 Ans: .

















.

where is an integrating constant.

**Problem-15:** **Exercise-12:**

 Ans:















whereis an integrating constant.

**Problem-15:****Exercise-13:**

Ans:











where is an integrating constant.

**Problem-17:**













where is an integrating constant.

***Method of substitution***

Sometimes, the integration of given integral  is relatively difficult. In this case, we can replace  by  and  by for integrating easily. This process is known as method of substitution.

***Illustrative Examples:***

**Problem-01**:  **Exercise-01**: 

 Ans: .

Put 



Now 







where is an integrating constant.

**Problem-02:** **Exercise-02:**

 Ans: .

Put 

Now 













where is an integrating constant.

**Problem-03:** **Exercise-03:**

 Ans: .

Put 

Now 





where is an integrating constant.

**Problem-04:** **Exercise-04:**

 Ans: .

Put 

Now 







where is an integrating constant.

**Problem-05:** **Exercise-05:**

 Ans: .



Put 

Now 





where is an integrating constant.

**Problem-06:**











where is an integrating constant.

**Problem-07:** **Exercise-06:**

 Ans: .

Put 

Now 









where is an integrating constant.

**Problem-08:** **Exercise-07:**

 Ans: .

Put 

Now 













where is an integrating constant.

**Problem-09:** **Exercise-08:**

 Ans: .

Put 

Now 







.

where is an integrating constant.

**Problem-10:**







Put 

Now 







where is an integrating constant.

**Problem-11:**











where is an integrating constant.

**Problem-12:** **Exercise-09:**

 Ans: 









where is an integrating constant.

**Problem-13:** **Exercise-10:**

 Ans: 









where is an integrating constant.

**Problem-14:**







Put 

Now 





where is an integrating constant.

**Problem-15:**











where is an integrating constant.

**Problem-16:** **Exercise-11:**

 Ans: .

Put 

Now 



















where is an integrating constant.

**Problem-17:**



Put 

Now 

















.

where is an integrating constant.

**Problem-18:**





Put 

Now 





where is an integrating constant.

**Problem-19:** **Exercise-12:**

 Ans: 









Put 

Now 





where is an integrating constant.

**Problem-20:**















Put 

Now 





.

where is an integrating constant.

**Problem-21:** **Exercise-13:**

 Ans: 



Now 











where, 

Put 

Now 





and

Put 

Now 





From (1) we have,



where is an integrating constant.

**Problem-22:** **Exercise-14:**

 Ans: 



Put 

or, 



Now 













where is an integrating constant.

***NOTE:*** Integrals of the type  can be evaluated exactly in the same way.

**Some Important Standard Integrals**

**Problem-01:** **Exercise-01:**

 Ans: .

 **Exercise-02:**

 Ans: .

 **Exercise-03:**

 Ans: .

where is an integrating constant.

**Problem-02:**

















where is an integrating constant.

**Problem-03:** **Exercise-04:**

 Ans: .

 **Exercise-05:**

 Ans: .





where is an integrating constant.

**Problem-04:** **Exercise-06:**

 Ans: .







.

where is an integrating constant.

**Problem-05:** **Exercise-07:**





















where is an integrating constant.

**Problem-06:** **Exercise-08:**











where is an integrating constant.

***NOTE:*** Integrals of the type can be evaluated exactly in the same way.

**Problem-07:** **Exercise-09:**

 Ans: .

Put 

Now 







.

where is an integrating constant.

**Problem-08:** **Exercise-10:**

 Ans: .

Put  ; 



Now 





.

where is an integrating constant.

**Problem-09:** **Exercise-11:**

 Ans: .

Put  ; 

 **Exercise-12:**

Now 

 ; putting









where is an integrating constant.

***NOTE:*** Integrals of the type  can be evaluated exactly in the same way.

**Problem-10:** **Exercise-13:**

 Ans: .

Put  or  **Exercise-14:** 

 Ans: .

Now **Exercise-15:** 

Ans: .







where is an integrating constant.

**Problem-11:** **Exercise-16:** 

 Ans: .

Put  or 



Now 









where is an integrating constant.

***NOTE:*** Integrals of the type  can be evaluated exactly in the same way.

**Problem-12:** **Exercise-17:** 

 Ans: .

Put 



Now 



Again let 

















where is an integrating constant.

***NOTE:*** Integrals of the type  can be evaluated exactly in the same way.

**Problem-13:** **Exercise-18:**

 Ans: .

Put  **Exercise-19:**

Now  Ans: .

 **Exercise-20:**





















.

where is an integrating constant.

***NOTE:*** Integrals of the type  can be evaluated exactly in the same way.

***Integration by Parts***

The formula for the integration of a product of two functions is referred to as integration by parts. *i.e,*

.

While applying the above rule for integration by parts to the product of two functions, care should be taken to choose properly the first function, i.e., the function not to be integrated.

**Illustrative Examples**:

**Problem-01:** **Exercise-01:**

 Ans: 

 **Exercise-02:** 

 Ans: 



where is an integrating constant.

**Problem-02:**























where is an integrating constant.

**Problem-03:** **Exercise-03:**

 **Ans:** 









where is an integrating constant.

**Problem-04:** **Exercise- 04:**

 **Ans:**













where is an integrating constant.

**Problem-05:** **Exercise- 05:**

 **Ans:**















where is an integrating constant.

**Problem-06:** **Exercise-06:**

 Ans: 

Put 

Now 











where is an integrating constant.

**Problem-07:** 



















where is an integrating constant.

**Problem-08:**



Put 

Now 











where is an integrating constant.

**Problem-09: Exercise-07: **

** Ans: .**

****

****

****

****

****

****

****

where is an integrating constant.

**Problem-10:  Exercise-08: **

 Ans: 





















where is an integrating constant.

***Integration of Trigonometric Functions***

**Problem-01:  Exercise-01: **

** Ans: **







Put ****

Now 





where c is an integrating constant.

**Problem-02:  Exercise-02: **

** Ans: **







Put 

Now 







where c is an integrating constant.

**Problem-03:  Exercise-03: **

** Ans: **

****

****

****

****

Put 

Now 











where c is an integrating constant.

**Problem-04: **

****

Put 

Comparing coefficient of ,  and constant terms, we get



Solving,



Now ****

****

****

****

where c is an integrating constant.

**Exercise-04: **

**Ans: **

**Exercise-05: **

**Ans: **